Sheet 4

1 (a) What is meant by the divergence of a vector field ?

(b) Derive an expression to explain (a)

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2 Determine the divergence of the following fields :

$(a)\ \bar{A} = x^2 \bar{a}_x + yz\, \bar{a}_y + xy\, \bar{a}_z$

$(b)\ \bar{A} = r \sin\phi\, \bar{a}_r + 2r \cos\phi\ \bar{a}_\phi + 2z^2\, \bar{a}_z$

$(c)\ \bar{A} = 5 \sin\theta\, \bar{a}_\theta + 5 \sin\phi\ \bar{a}_\phi$ at $\left(0.5, \frac{\pi}{4}, \frac{\pi}{4}\right)$

$$\left[\begin{array}{c} \nabla.\bar{A} = 2x + z \\ \nabla.\bar{A} = 4z \\ \nabla.\bar{A}\big|_{\left(0.5,\frac{\pi}{4},\frac{\pi}{4}\right)} = 24.142 \end{array}\right]$$

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3 Show that the vector field $\bar{F} = e^{-y}\left(\cos x\, \bar{a}_x - \sin x\, \bar{a}_y\right)$ solenoidal . Explain from the point of view of source and sink existence .

$$[\nabla.\bar{F} = 0]$$

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4 If the electric field $\bar{E} = y\bar{a}_x + x\bar{a}_y$, show that the given region does not contain any electric charge .

$$[\nabla.\bar{E} = 0]$$

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5 Determine the net flux of the vector field $\bar{F} = r\, \bar{a}_r + \bar{a}_\phi + z\, \bar{a}_z$ leaving a cylindrical closed surface defined by $r = 1$, $0 \le \phi \le \pi$ and $0 \le z \le 1$. Then verify the divergence theorem .

$$\left[\frac{3\pi}{2}\right]$$

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6 Given that

$$\bar{D} = \left(\frac{10r^3}{4}\right)\bar{a}_r$$

in cylindrical coordinates , evaluate both sides of the divergence theorem for the volume enclosed by $r = 1m$, $r = 2m$, $z = 0$ and $z = 10m$

$[\mathbf{750\,\pi}]$

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7 If $\bar{F} = \left(\frac{5r^2}{4}\right)\bar{a}_r$ in a spherical coordinates . Verify the divergence theorem for the volume enclosed by $r = 4\text{m}$ and $\theta = \frac{\pi}{4}$

$\left[\mathbf{640\,\pi}\left[\mathbf{1} - \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right] = \mathbf{588.89}\right]$

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8 Verify the divergence theorem , if $\bar{F} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$ for a cube with sides of 2m and its center is $(1,1,1)$

$[\mathbf{24}]$